

# A Doppler Shift model to explain the Cyclotron Line Variability in X-Ray Pulsars

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## ABSTRACT

A simple model to explain the phase-dependence of the cyclotron absorption line observed in many X-Ray pulsars is presented. It includes several relativistic effects, namely gravitational redshift, gravitational light deflection, and – most important – doppler shift of the photon energy in the infalling plasma. It is shown that previous estimates of neutron star magnetic fields neglecting the last effect give results which are too small by about a factor of two. Finally the developed equations are used to determine the neutron star radius and magnetic field of Her X-1, but the large uncertainties in the geometry prevent the results from seriously constraining the parameters.

*Subject headings:* line: formation — relativity — stars: magnetic fields — stars: neutron — X-rays: binaries

## 1. Introduction

Cyclotron absorption lines are a important diagnostic tool in the analysis of X-Ray pulsars since they provide a way to measure the magnetic field of the neutron star directly. First discovered in Her X-1 (Trümper et al. 1978), there are now about a dozen systems in which they can be seen (Makishima et al. 1999).

In the last 20 years, phase-resolved spectroscopy of the cyclotron line region has become possible and has been done for several systems (Her X-1: Voges et al. (1982); Soong et al. (1990), 4U 1538-52: Clark et al. (1990), 4U 1907+09, Vela X-1: Makishima et al. (1999), Cen X-3: Burderi et al. (2000)). In most of them the cyclotron line energy varies with the pulse phase within 10 to 20%.

Model calculations of radiative transfer in slabs and columns by several authors yielded viewing-angle dependent cyclotron line features, e. g. Mészáros & Nagel (1985); Isenberg et al. (1998). However, the line variations obtained there are not sufficient to explain the observed values. Brainerd & Mészáros (1991) find that absorption in the infalling plasma of an accretion column yields

a much stronger feature at lower energy than the one obtained in the slab models, the energy of which is angle-dependent due to the bulk motion of the infalling plasma. A second consequence of the high velocity (the free-fall velocity is about  $0.5c$  at the neutron star surface!) is that most of the scattered photons are directed back onto the neutron star, so the line is seen in absorption.

In the following section, a simple analytic model that gives an analytic formula for the cyclotron line energy seen by the observer will be introduced. After discussing some of its properties, the model is applied to Her X-1 where the geometrical parameters have been analyzed (Blum & Kraus 2000). Finally, the results and implications are discussed.

## 2. A simple model

Consider a neutron star with radius  $R_N$  and gravitational (Schwarzschild) radius  $R_S$  in the Schwarzschild metric. Assume that the region around the magnetic poles where the accreted plasma hits the neutron star is small, so that  $B \approx \text{const}$  and the direction of the infalling matter is orthogonal to the neutron star surface; the polar cap itself is assumed to be plane. Quantities in the Lorentz system resting on the neutron star surface are marked with a hat in the following, whereas the ones in the rest system of the infalling plasma have an index 0.

Neglecting radiation pressure, the local free-fall velocity on the surface is  $\hat{\beta}_{\text{ff}} = \hat{v}_{\text{ff}}/c = \sqrt{R_S/R_N}$ . If the matter is decelerated above the surface, a factor  $\eta \leq 1$  can be introduced so that  $\hat{\beta} = \eta \hat{\beta}_{\text{ff}}$ .

In the nonrelativistic limit ( $B \ll B_{\text{crit}} \approx 4.4 \times 10^9 \text{T}$ ), the cyclotron energy in the frame at the neutron star surface is

$$\hat{E}_c = \hbar \omega_c = \frac{\hbar e B}{m_e} = 11.58 B_8 \text{ keV} \quad (1)$$

where  $B_8$  is the magnetic field in  $10^8$  Tesla.

In addition to the magnetic field strength at the neutron star surface, there are several effects that determine at which energy the cyclotron absorption line in the spectrum is seen by a distant observer: gravitational redshift, gravitational light bending, and the doppler effect due to the bulk motion of the infalling plasma.

The first one of them, gravitational redshift, simply shifts the spectrum to a lower energy by a factor of  $\sqrt{1 - R_S/R_N}$  (and decreases the intensity, which is of no interest here). However, the radius of the neutron star is generally not known, so that results of theoretical works (e. g. Shapiro & Teukolsky (1983); Mészáros & Riffert (1987); Wu et al. (1991)), models fits (Bulik et al. 1995) or other indirect measurements (Zhang et al. 1997) have to be used. Typical values range from 2 to about  $4 R_S$ .

Gravitational light deflection changes the direction under which radiation emerging from the hot spot is seen. Again, the strength of this effect depends on the neutron star radius. Its effect can be described by an elliptic integral (see e.g. Weinberg (1972)):

$$\Delta\theta = \int_{R_N}^{\infty} \frac{b}{r^2} \frac{dr}{\sqrt{1 - A(r)b^2/r^2}}, \quad (2)$$

where  $A(r) = 1 - R_S/r$  and  $b$  is the impact parameter of the photon, which is given by  $b = R_N \sin \hat{\vartheta} / \sqrt{1 - R_S/R_N}$ .  $\hat{\vartheta}$  is the propagation angle of the photon with respect to the radial direction at the surface.

The third effect, doppler shift, is responsible for the angle- (and thus phase-) dependence of the line energy: The resonance occurs (for cold plasma) when the photon energy equals the cyclotron energy *in the plasma rest frame*. While the magnetic field strength is not altered by the Lorentz boost as long as the motion of the matter is along the field lines, the photon energy is:

$$E_0 = \hat{E} \frac{(1 + \cos \hat{\vartheta} \hat{\beta})}{\sqrt{1 - \hat{\beta}^2}} \quad (3)$$

In summary, the spectrum of the radiation going in the direction  $\hat{\vartheta}$  at the neutron star surface will be seen by an observer at  $\hat{\vartheta} + \Delta\theta$ ; the absorption line will appear at

$$E_{c,obs}(\theta_{obs}) = \frac{\hbar e B}{m_e} \frac{\sqrt{1 - (\eta \hat{\beta}_{ff})^2}}{(1 + \cos \hat{\vartheta} \eta \hat{\beta}_{ff})} \sqrt{1 - R_S/R_N} \stackrel{\eta=1}{=} \frac{\hbar e B}{m_e} \frac{1 - R_S/R_N}{(1 + \cos \hat{\vartheta} \sqrt{R_S/R_N})} \quad (4)$$

Figure 1 shows the measured cyclotron line energy in units of the line energy in the plasma rest frame for a neutron star of  $R_N/R_S = 3$ . Note that for this ratio, the line stops at the angle  $\theta_{obs} \approx 120^\circ$ , because photons would have to pass through the neutron star in order to get there. However, in more sophisticated models, photons can be scattered at some point above the hot spot and reach these regions.

Several more curves are shown in the figure: The constant one is the line one gets neglecting the doppler effect, which has been done in most publications computing the magnetic field in X-Ray binaries so far. The dotted lines are without gravitational redshift or gravitational light deflection; the crosses are the results of a Monte-Carlo-Simulation that uses a realistic geometry and nonrelativistic cold cross sections. Details of this model will be published elsewhere (Weth et al., in preparation). These line energies are systematically lower than the ones of the simple model; the reason for that is that scattering takes place also above the hot spot, where the magnetic field drops steeper than  $r^{-3}$  (Wasserman & Shapiro 1983). The difference in the plot corresponds to a height of about 150m.

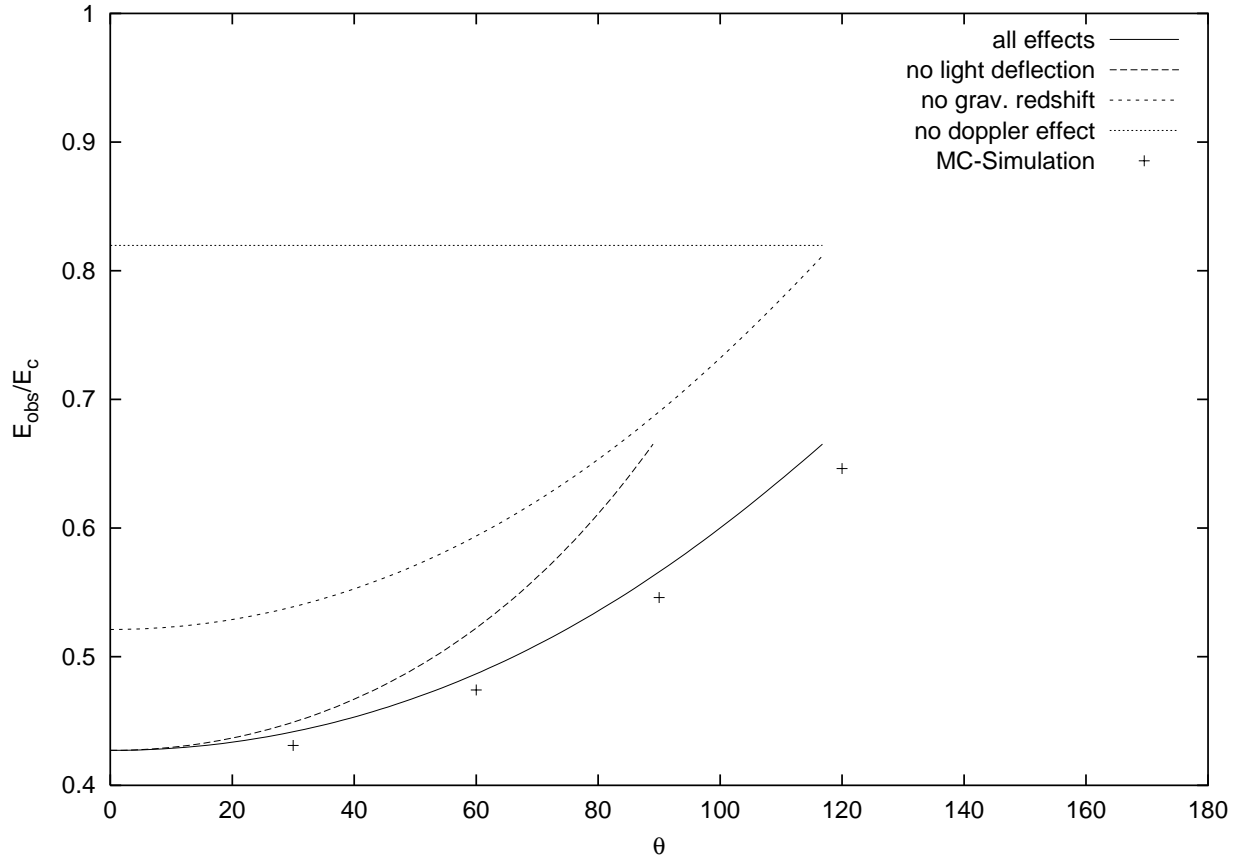


Fig. 1.— Cyclotron line energy seen by the observer in units of the cyclotron energy on the neutron star surface (solid line). The other curves demonstrate how neglecting one of the relativistic effects changes the result. The crosses are the result of a Monte-Carlo-Simulation of a more sophisticated model. The neutron star radius is  $3R_S$ ; no radiation pressure is included.

In real systems, there are two accreting polar caps, so what the observer sees is the sum of two spectra at different angles, which are  $\theta_1$  and  $\theta_2 = 180^\circ - \theta_1$  in the simplest case. Because the luminosity depends on the direction, too, at some angles the line feature of one cap will be hidden in the background of the other one. In most cases however, these two lines will be too close to each other to be resolved.

### 3. Application to Her X-1

A quantity which can be measured is

$$\xi(R_N/R_S) = \frac{E_{c,\max}}{E_{c,\min}} \quad (5)$$

Its value is always smaller than the theoretical maximum which can be obtained by taking the cyclotron energies seen at  $\theta_{\text{obs}} = 0$  and  $\theta_{\text{obs}} = 180^\circ$ . It depends only on the neutron star radius and on  $\eta$ . However, this does not constrain the parameter  $R_N/R_S$  significantly unless  $\xi \geq 2$ , which has not been found yet in any system.

In the case of Her X-1 the geometry (i. e. inclination of the rotation axis and position of the magnetic poles) is known (Blum & Kraus 2000), so a model value of  $\xi$  can be computed by using the appropriate angles. Unfortunately Her X-1 is not ideal for our purpose because of its high luminosity of  $L_{37} = 2.0$  (Nagase 1989), so that radiation pressure cannot be neglected here. In consequence, there are three unknown parameters ( $R_N$ ,  $B$ , and  $\eta$ ), but only two quantities that can be measured ( $E_{c,\max}$  and  $E_{c,\min}$ ).

In order to compute the parameters, an additional assumption is needed. Two approaches will be tried: First  $R_N/R_S$  is fixed to a specific value, in the second one radiation pressure is completely neglected by setting  $\eta = 1$ .

Adopting  $E_{\max} = 36.9 \pm 0.3$  keV,  $E_{\min} = 30.3 \pm 1.0$  keV (Soong et al. 1990) and  $i = 83^\circ \pm 4^\circ$  and  $\theta_{\text{pole}} = 20^\circ \pm 10^\circ$  (Blum & Kraus 2000), and further assuming a neutron star radius of  $R_N = 3.0 R_S$ , equation (4) yields  $\eta = 0.9 \pm 0.5$ , which is consistent with no radiation pressure. Most of the error is due to the large uncertainty in  $\theta_{\text{pole}}$ , the positions of the magnetic poles relative to the rotation axis.

Fixing  $\eta = 1$  and determining the neutron star radius gives the right range ( $2.4 \leq R_N/R_S \leq 7.5$ ), but including the error in  $\xi$  causes the upper limit to be shifted beyond  $12 R_S$  (Fig. 2).

Finally, the magnetic field at the neutron star surface is computed from the previous results ( $\eta = 0.9 \pm 0.5$ ,  $R_N = 3.0 R_S$  fixed). This yields  $B_8 = 5.0 \pm 0.9$ , the main cause for the large error being the uncertainty in  $\eta$ . Varying  $R_N$  changes the result by about  $\pm 1$ . This value of  $B$  is considerably higher than the one given by Voges et al. (1982), which was 3.4.

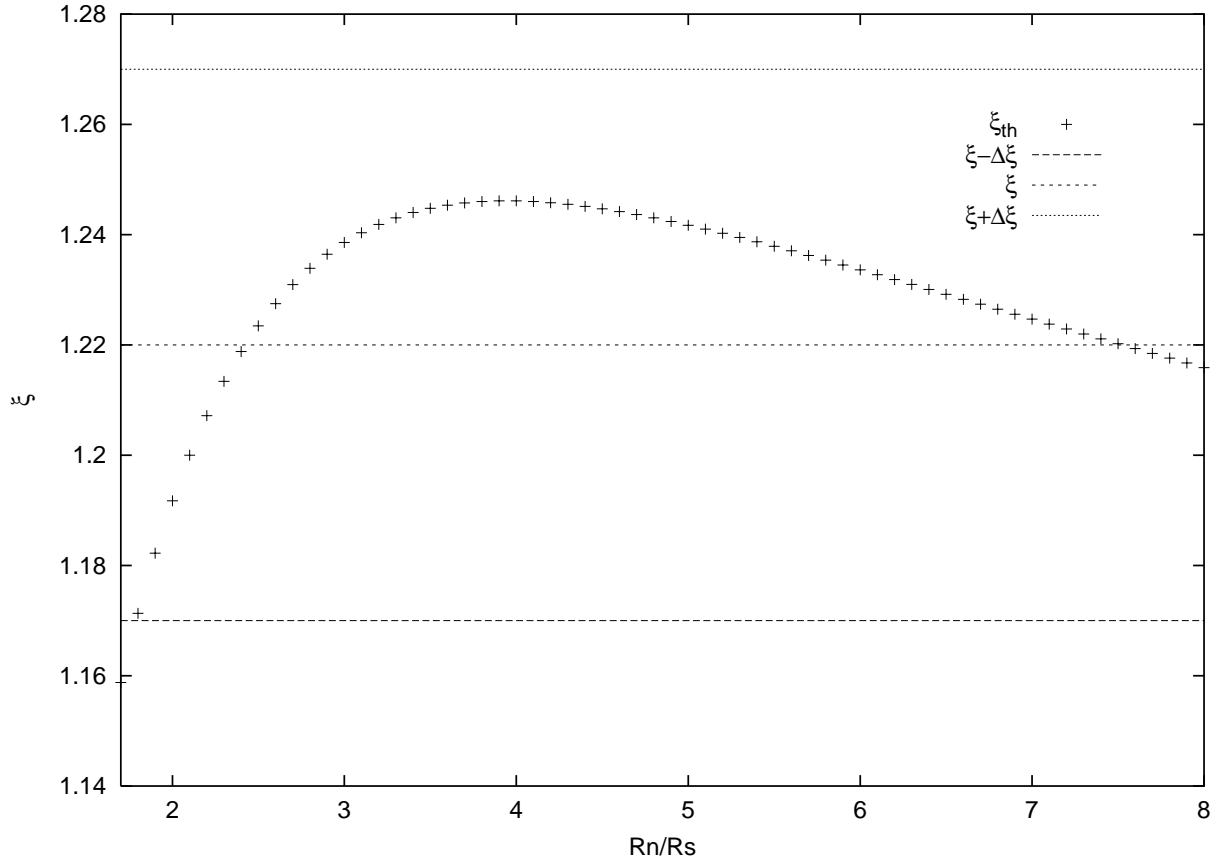


Fig. 2.—  $\xi(R_N/R_S)$  without radiation pressure. The horizontal lines are the measured  $\xi$  and the error range.

#### 4. Discussion

With the doppler shift in the accretion column, a simple explanation for the phenomenon of the phase-dependent cyclotron line energy was found; it should occur in almost every system except those in which the radiative pressure is so high that the velocity of the infalling matter becomes small compared to  $c$ . However, there are some problems: Cyclotron line absorption might not only take place close to the hot spot, but also higher in the accretion column, especially when the column is optically thick; this shifts the cyclotron line centers to lower energies. In this case, not the neutron star radius  $R_N$  is obtained, but the distance of the emission region from the neutron star center (which is at least an upper limit to the radius). Another problem is that not the full range of cyclotron line energies might be seen - if the line feature from one cap falls into a region with high continuum from the other one, it will be hard to detect.

Still, the range of values obtained for the neutron star radius is where one expects the actual radius to be. The resulting magnetic field strength is significantly higher than the usual estimates ( $B_8 \approx 5$  instead of 3), which only take into account gravitational redshift but not the doppler effect due to the moving plasma. This fact was already mentioned by Brainerd & Mészáros (1991).

A prediction of this simple model is the presence of two absorption lines (one from each emission region) in the spectrum that move anticyclically with the pulse phase. This should make them distinguishable from a second harmonic, the energy shift of which is in phase with the one of the fundamental line. The detection of absorption features from both hot spots will only be possible for systems where the angle between magnetic and rotation axis is large, so that one can see the emission regions at all angles; in other systems, they will be too close to each other to be resolved.

On the other hand, for not too large neutron star radii, the absorption feature from the surface and the one from the column should be distinct (Brainerd & Mészáros 1991). However, it is not clear if the surface feature is visible at all, since it might be filled up by photon spawning effects (Isenberg et al. 1998). The column absorption feature is probably not changed too much by those, because most scattered photons are advected back onto the neutron star.

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